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On the New Integral Transform "ELzaki Transform" Fundamental Properties Investigations and Applications

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Abstract

ELzaki transform, whose fundamental properties are presented in this paper, is still not widely known, nor used, ELzaki transform may be used to solve problems without resorting to a new frequency domain. In this paper we provide ELzaki transform for a more comprehensive list of functions. Aside from a paradigm change into the thought process of ELzaki transform usage with respect to applications to differential equations, we introduce more general shift theorems .Moreover; we use laplace-ELzaki duality (LED) to invoke a complex inverse ELzaki transform, as a Bromwhich contour integral formula.

Keywords: Elzaki Transform- Differential Equations-Laplace transform-Applications.

Introduction

Due to its simple formulation and consequent special and useful properties,. It has revealed herein and else where that it can help to solve intricate problems in engineering mathematics and applied science.

However, despite the potential presented by this new operator, only few theoretical investigations have appeared in the literature.

Most of the available transform theory books, if not all, do not refer to ELzaki transform. On the other hand, for historical a accountability, we must note that a related formulation, called s-multiplied Laplace transform, was announced as early as 1948 if not before, Tarig M.ELzaki and Salih M.ELzaki papers [1,2,3,4,5,6], showed ELzaki transform applications to partial differential equations ordinary differential equations, system of ordinary and partial differential equations and integral equations.

Also Tarig M.ELzaki showed that ELzaki transform can be effectively used to solve ordinary differential equations [1] and engineering control problems. One more, Tarig work was followed by Tarig and Salih, introduce a complex inversion formula for ELzaki transform (in this paper).Further more Tarig.M.ELzaki extended this transform method to variables with emphasis on Solutions to partial differential equations.

Salih0 M.ELzaki presented applications to convolution type integral equations with focus on production problems and Laplace –Elzaki duality was highlighted, and used to establish or corroborate many fundamental useful properties of this new transform. A table of transform of some basic functions was provided.

The Discrete ELzaki Transform

A new integral transform called ELzaki transform [1] defined for functions of exponential order, is proclaimed. We consider functions in the set A defined by,

$$A=\left\{ f\left(t\right)\middle|\exists M, k_{1,} k_2>0\middle| f\left(t\right)<M\, e^{\frac{|t|}{k_j}}, if\ t\in(-1)^j \mathrm{X}[0,\infty)\right\} \tag{2-1}$$

ELzaki transform is defined by:

$$E\left[f\left(t\right)\right]=u^2\int_0^\infty f\left(ut\right)e^{-t}dt=T(u), \qquad u\in\left(k_1,k_2\right) \tag{2-2}$$

ELzaki transform was shown to have unite preserving properties, and hence may be used to solve problems without resorting to the frequency domain .As will be seen below , this is one of the advantages of this new transform, particularly in applications to problems with physical dimensions .In fact ELzaki transform which is itself linear preserves linearity.

Theorem (2-1)

ELzaki transform amplifies the coefficients of the power series function,

$$f\left(t\right)=\sum_{n=0}^{\infty}a_n t^n \tag{2-3}$$

Is

$$\mathrm{E}\left[f(t)\right]=T(u)=\sum_{n=0}^{\infty} n!\, a_n\, u^{n+2} \tag{2-4}$$

Proof

Let $f(t)$ be in A. If $f(t)=\sum_{n=0}^{\infty} a_n t^n$, then by Taylor's function expansion theorem

$$f(t)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)t^n}{n!} \tag{2-5}$$

Therefore by (2-2) we have:

$$\mathrm{E}\left[f(t)\right]=u^2\int_0^{\infty}\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}(ut)^n e^{-t}\,dt=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)u^{n+2}}{n!}$$

$$\int_0^{\infty} t^n e^{-t}dt=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}u^{n+2}\,\Gamma(n+1)$$

Then

$$\mathrm{E}\left[f(t)\right]=\sum_{n=0}^{\infty} f^{(n)}(0)\, u^{n+2} \tag{2-6}$$

Consequently,

$$\begin{aligned}\mathrm{E}\left[(1+t)^m\right] &= \mathrm{E}\sum_{n=0}^{m} C_n^m\, t^n = \mathrm{E}\sum_{n=0}^{m}\frac{m!}{n!(m-n)!}\,t^n\\ &=\sum_{n=0}^{m}\frac{m!}{(m-n)!}u^{n+2}=\sum_{n=0}^{m} P_n^m\, u^{n+2}\end{aligned} \tag{2-7}$$

ELzaki transform sends combinations, C_n^m, into permutations, P_n^m, and hence may incur more order into discrete systems.

Also $\mathrm{E}\left[f(t)\right]$ converges, in an interval containing $u=0$, provided that the following conditions are satisfied.

$$\begin{aligned}&(i)\ \ f^{(n)}(0)\to 0 \ \ as \quad n\to\infty\\ &(ii)\ \ \lim_{n\to\infty}\left|\frac{f^{(n+1)}(0)}{f^{(n)}(0)}u\right|<1\end{aligned} \tag{2-8}$$

This means that the convergence radius r of $\mathrm{E}\left[f(t)\right]$, depends on the sequence $f^{(n)}(0)$, since

$$r = \lim_{n\to\infty}\left|\frac{f^{(n)}(0)}{f^{(n+1)}(0)}\right| \tag{2-9}$$

For example, consider the function

$$f(t)=\begin{cases}\ln(t+1) \ , & t\in(-1,1)\\ 0 \ , & other \ \ wise\end{cases} \tag{2-10}$$

Since $f(t)=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{t^{n}}{n}$, then except for $u=0$

$$\mathrm{E}[f(t)]=\sum_{0}^{\infty}(-1)^{n-1}(n-1)!\,u^{n+2} \tag{2-11}$$

Diverges throughout, because its convergence radius.

$$r=\lim_{n\to\infty}\left|\frac{(-1)^{n-1}(n-1)!}{(-1)^{n}\ n!}\right|=\lim_{n\to\infty}\frac{1}{n}=0 \tag{2-12}$$

Corollary (2-2)
Up to null functions, the inverse discrete ELzaki transform. $f(t)$, of the power series.

$T(u)=\sum_{n=0}^{\infty}b_n u^{n+2}$ Is given by

$$\mathrm{E}^{-1}[T(u)]= \quad f(t)=\sum_{n=0}^{\infty}\frac{b_n}{n!}\ t^{n} \tag{2-13}$$

In the next section, we provide a general inverse transform formula, albeit in a complex setting.

Laplace – Elzaki Duality and the Complex ELzaki Inversion Formula

Tarig M. ELzaki showed that ELzaki transform is the dual of Laplace transform. Hence, one should be able to rival it to a great extent in problem solving.

Defined for $\mathrm{Re}(s)>0$, the Laplace transform is given by.

$$F(s)=L[f(t)]=\int_{0}^{\infty}e^{-st}f(t)dt \tag{3-1}$$

According to definition (3-1) ELzaki and Laplace transforms exhibit a duality, relation expressed as follows

$$T(u) = u F\left(\frac{1}{u}\right) \quad , \quad F(s) = sT\left(\frac{1}{s}\right) \tag{3-2}$$

Equations (3-2), which in the sequel shall be referred to as $(L\ E\ D)$, which is the short for Laplace – ELzaki duality.

Examples:

(1) $L[\sin t] = \dfrac{1}{s^2+1}$ $\quad (2)$ $L\left[e^{at}\right] = \dfrac{1}{s-a}$ $\quad (3)$ $L\left[J_0(at)\right] = \dfrac{1}{\sqrt{s^2+a^2}}$,

J_0 ,is Bessel's function by using $(L\ E\ D)$ we obtain ELzaki transform of the above functions as follows.

$$(1)\ \mathrm{E}[\sin t] = u\left[\frac{1}{\frac{1}{u^2}+1}\right] = \frac{u^3}{1+u^2} \tag{3-3}$$

$$(2)\ E\left[e^{at}\right] = u\left[\frac{1}{\frac{1}{u}-a}\right] = \frac{u^2}{1-au} \tag{3-4}$$

$$(3)\ \mathrm{E}\left[J_0(at)\right] = \frac{u}{\sqrt{\frac{1}{u^2}+a^2}} = \frac{u^2}{\sqrt{1+a^2u^2}} \tag{3-5}$$

This also consistent with the established differentiation and integration formulas

$$\mathrm{E}\left[f'(t)\right] = \frac{T(u)}{u} - uf(0) \tag{3-6}$$

$$\mathrm{E}\left[\int_0^t f(\tau)d\tau\right] = u\mathrm{E}\left[f(t)\right] \tag{3-7}$$

The $(L\ E\ D)$ is further demonstrated by the following example,

$$\frac{dx}{dt} + x = 1 \quad , \quad x(0) = 0 \tag{3-8}$$

We traditionally resort to the Laplace transform to form the auxiliary equation,

$F(s)(s+1) = \dfrac{1}{s}$ From which we get: $F(s) = \dfrac{1}{s(s+1)} = \dfrac{1}{s} - \dfrac{1}{s+1}$

And then invert to find the solution

$$x(t)=1-e^{-t} \tag{3-9}$$

Alternatively, using ELzaki transform of (3-8) we get:

$\dfrac{T(u)}{u}+T(u)=u^2$, Which exhibits the transform $T(u)$,

$$T(u)=\frac{u^3}{1+u}=u^2-\frac{u^2}{1+u} \tag{3-9) a}$$

Inverting to get the solution: $x(t)= \ E^{-1}\left[T(u)\right]= \ 1-e^{-t}$

The solution of differential equations in applied mathematics requires the inverse transform. Luckily, the $(L\,E\,D)$ in (3-2) helps us to establish such a useful tool. Indeed, by virtue of Cauchy theorem, and the residue theorem, the following is a Bromwich contour integration formula for the complex inverse ELzaki transform.

Theorem (3-1)

Let $T(u)$ be ELzaki transform of $f(t)$ such that.

(i) $sT\left(\dfrac{1}{s}\right)$ is a meromorphic function, with singularities having $\operatorname{Re}(s)<\alpha$, and

(ii) There exists a circular region Γ with radius R and positive constants, M and K with.

$$\left|sT\left(\frac{1}{s}\right)\right|<MR^{-K} \tag{3-10}$$

Then the function $f(t)$, is given by

$$\mathrm{E}^{-1}\left[T(s)\right]=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} e^{st}sT\left(\frac{1}{s}\right)ds=\sum residues\ \ of\ \ \left[e^{st}sT\left(\frac{1}{s}\right)\right] \tag{3-11}$$

Proof:

Let $F(S)=L\left[f(t)\right]$, and $T(u)=\mathrm{E}\left[f(t)\right]$ be Laplace and ELzaki transform of $f(t)$, respectively then by the complex inversion formula for the Laplace transform for $t>0$, the function $f(t)$ is given by,

$$f(t)=L^{-1}\left[F(s)\right]=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} e^{st}F(s)ds \tag{3-12}$$

Where $s = x + iy$ a complex variable .This includes branch points, essential singularities, and poles.

By virtue of the residue theorem, we have,

$$f(t) = L^{-1}[F(s)] = \frac{1}{2\pi i}\int_{\alpha - i\infty}^{\alpha + i\infty} e^{st}F(s)ds = \sum res\left[e^{st}F(s)\right] \tag{3-13}$$

By invoking into the previous relation, of $(L\,E\,D)$ between the transform F and T namely that $sT\left(\frac{1}{s}\right)$, we get the desired conclusion of theorem (3-1).

Obviously, the previous theorem can be readily applied to the function in (3-9) a to get the solution in (3-9).Indeed one easily recognizes that the residues of $\frac{e^{st}}{s(s+1)}$ do occur at the poles $s = -1$ and $s = 0$ with respective values $-e^{-t}$ and 1.

ELzaki Theorems for Multiple Differentiation, Integration and convolution

The next theorem was proved by virtue of the $L\,E\,D$ between ELzaki and Laplace transforms .In this paper we use an induction argument to prove the result.

Theorem (4-1)

Let $f(t)$ be in A and Let $T_n(u)$ denote ELzaki transform of nth derivative, $f^{(n)}(t)$ of $f(t)$, then for $n \geq 1$,

$$T_n(u) = \frac{T(u)}{u^n} - \sum_{k=0}^{n-1} u^{2-n+k} f^{(k)}(0) \tag{4-1}$$

Proof:

For $n = 1, (3-6)$ shows that (4-1) holds. To proceed to the induction step, we assume that (4.1) holds for n and prove that it carries to $n+1$. Once more by virtue of (3-6) we have,

$$\begin{aligned} T_{n+1}(u) &= \mathrm{E}\left[f^{(n)}(t)\right]' = \frac{\mathrm{E}\left[f^{(n)}(t)\right]}{u} - u f^{(n)}(0) \\ &= \frac{T_n(u)}{u} - u f^{(n)}(0) = \frac{T_n}{u^{n+1}} - \sum_{k=0}^{n} u^{1-n+k} f^{(k)}(0) \end{aligned} \tag{4-2}$$

In particular, this means that ELzaki transform, $T_2(u)$, of the second derivative of the function, $f(t)$, is given by:

$$T_2(u) = \mathrm{E}\left[f''(t)\right] = \frac{T(u)}{u^2} - f(0) - uf'(0) \tag{4-3}$$

For instance, the general solution of the second order equation,

$$\frac{d^2y}{dt^2} + w^2 y = 0 \tag{4-4}$$

With the initial conditions: $y(0) = a \quad , \quad y'(0) = b$

Can easily be transformed into its ELzaki transform, equivalent.

$$\frac{T(u)}{u^2} - y(0) - uy'(0) + w^2 T(u) = 0 \tag{4-5}$$

With general ELzaki solution:

$$T(u) = \frac{au^2 + bu^3}{1 + w^2u^2} = \frac{au^2}{1 + w^2u^2} + \frac{bu^3}{1 + w^2u^2} \tag{4-6}$$

And upon inverting by using theorem (3-1) we get the general time solution:

$$y(t) = a\cos(wt) + \frac{b}{w}\sin(wt) \tag{4-7}$$

Obviously, theorem (4-1), shows that Elzaki transform can be used just like Laplace transform, as in the previous example, to solve both linear differential equations of any order. The next theorem allows us to use ELzaki transform to solve differential equations involving multiple integrals of the dependent variable as well, by rendering then into algebraic ones.

Theorem (4-2)

Let $f(t)$ be in A , and let $T^n(u)$ denote ELzaki transform of the nth antiderivative of $f(t)$, obtained by integrating the function, $f(t)$, n times successively.

$$w^n(t) = \int\int_0^t \ldots \int_0^t f(\tau)\,(d\tau)^n \tag{4-8}$$

Then for $n \geq 1$

$$T^n(u) = \mathrm{E}\left[w^n(t)\right] = u^n T(u) \tag{4-9}$$

Proof:

For $n = 1$, (3-7) shows that (4-9) holds, we assume that (4.9) holds for $n,$ and prove that it carries to $n+1.$ Once more, by virtue of $(3.7),$ we have

$$T^{n+1}(u)=\mathrm{E}\left[w^{n+1}(t)\right]=\mathrm{E}\left[\int_0^t w^n(\tau)\,d\tau\right]= \\ u\mathrm{E}\left[w^n(t)\right]=u\left[u^n T(u)\right]=u^{n+1}T(u) \tag{4-10}$$

This theorem generalizes ELzaki convolution theorem (4.1). Tarig M.ELzaki states that the transform of. $(f*g)(t)=\int_0^t f(\tau)g(t-\tau)d\tau$ (4-11)

Is given by,

$$\mathrm{E}\left[(f*g)(t)\right]=\frac{1}{u}F(u)G(u) \tag{4-12}$$

Corollary (4-3)

Let $f(t), g(t), h(t), h_1(t)h_2(t)...and\ h_n(t)$ be functions in A, having ELzaki transforms, $F(u)$, $G(u)$, $H(u)$, $H_1(u)$, $H_2(u)....,and\ H_n(u)$, respectively, then ELzaki transform of

$$(f*g)^n(t)=\int_0^t\int_0^t....\int_0^t f(\tau)g(t-\tau)(d\tau)^n \tag{4-14}$$

Is given by: $\mathrm{E}\left[(f*g)^n(t)\right]=\frac{1}{u^n}F(u)\ G(u)$. Moreover, for any integer $n\geq 1$,

$$\mathrm{E}\left[(h_1*h_2*...h_n)(t)\right]=u^{n-1}H_1(u)H_2(u)...H_n(u) \tag{4-15}$$

Proof:

Equation (4.14) is just a direct consequence of theorem (4.2) due to the property of associability of the convolution operator, (4.12) that implies (4-15).

The previous results can be used in powerful manner to solve integral and integral-differential equations.

ELzaki Transform Multiple Shift Theorem

The discrete ELzaki transform can be used effectively to discern some rules on how the general transform affects various functional operations, Tarig M.ELzaki prove that

$$\mathrm{E}\left[tf'(t)\right]=u^2\frac{d}{du}\left[\frac{T(u)}{u}-uf(0)\right]-u\left[\frac{T(u)}{u}-uf(0)\right] \tag{5-1}$$

And that:

$$\mathrm{E}\left[te^t\right]=\frac{u^3}{(1-u)^2} \tag{5-2}$$

One may ask how ELzaki transform acts on $t^n f(t)$. clearly, if $f(t)=\sum_{n=0}^{\infty} a_n t^n$, then

$$\mathrm{E}\left[tf(t)\right]=\sum_{n=0}^{\infty}(n+1)!a_n u^{n+3} = u^3\frac{d}{du}\sum_{n=0}^{\infty} n!a_n u^{n+1} \tag{5-3}$$

Theorem (5-1)
Let $T(u)$ be ELzaki transform of the function $f(t)$ in A, then ELzaki transform of the function $tf(t)$ is given by:

$$\mathrm{E}\left[tf(t)\right]=u^2\frac{d}{du}T(u)-uT(u) \tag{5-4}$$

Proof:
The function $t f(t)$ is in A, since $f(t)$ is so: and integrating by parts we find that.

$$\frac{d}{du}T(u)= T'(u) =\frac{d}{du}\int_0^{\infty} u e^{-\frac{t}{u}} f(t)dt = \int_0^{\infty}\frac{\partial}{\partial u}\left[ue^{-\frac{t}{u}} f(t)\right]dt$$

$$= \int_0^{\infty}\frac{1}{u}e^{-\frac{t}{u}}\left(t f(t)\right)dt+\int_0^{\infty} e^{-\frac{t}{u}} f(t)\ dt= \frac{1}{u^2}\mathrm{E}\left[t\ f(t)\right]+\frac{1}{u}\mathrm{E}\left[f(t)\right]$$

Then we have:

$$\mathrm{E}\left[t\ f(t)\right]= u^2\frac{d}{du}T(u)-uT(u)$$

In the general cases we can external theorem (5.1) as,

$$(i)\ \mathrm{E}\left[t f'(t)\right]=u^2\frac{d}{du}\left[\frac{T(u)}{u}-uf(0)\right]-u\left[\frac{T(u)}{u}-uf(0)\right]$$

$$(ii)\ \mathrm{E}\left[t^2 f'(t)\right]=u^4\frac{d^2}{du^2}\left[\frac{T(u)}{u}-uf(0)\right]$$

$$(iii)\ \mathrm{E}\left[t f''(t)\right]=u^2\frac{d}{du}\left[\frac{T(u)}{u^2}-f(0)-uf'(0)\right]-u\left[\frac{T(u)}{u^2}-f(0)-uf'(0)\right]$$

$$(iv)\ \mathrm{E}\left[t^2 f''(t)\right]= u^4\frac{d^2}{du^2}\left[\frac{T(u)}{u^2}-f(0)-uf'(0)\right]$$

The proof of these equations is easy, by using theorem (5.1)

Application to Bessel Equation

Consider the Bessel equation,

$$t\,y'' + y' + a^2 ty = 0 \quad , \quad y(0) = 1 \tag{6-1}$$

Applying ELzaki transform yields,

$$u^2 \frac{d}{du}\left[\frac{T(u)}{u^2} - 1 - uc\right] - u\left[\frac{T(u)}{u^2} - 1 - uc\right] + \frac{T(u)}{u} - u + a^2\left[u^2 \frac{d}{du}T(u) - uT(u)\right] = 0 \tag{6-2}$$

Where $c = y'(0)$. (6-2) can be written in the form:

$$T'(u) - \frac{2T(u)}{u} + a^2u^2T'(u) - a^2uT(u) = 0 \text{, Or}$$

$$\frac{T'(u)}{T(u)} = \frac{2}{u} - \frac{a^2u}{1+a^2u^2} \tag{6-3}$$

Integrating bath side of (6-3) we get:

$$T(u) = \frac{Au^2}{\sqrt{1+a^2u^2}}$$

Where A is a constant of integration. By the inverse ELzaki transform, we obtain $y(t) = A\,J_0(at)$

Conclusion

Generalizations of all existing ELzaki differentiation, integration and convolution theorems in the existing literature are demonstrated and so also generalizing all existing ELzaki shifting theorems. The Laplace – Elzaki duality $(L\,E\,D)$ will be used to invoke a complex inverse ELzaki transform.

Appendix

ELzaki Transform of some Functions

$f(t)$	$\mathrm{E}[f(t)]=T(u)$
1	u^2
t	u^3
t^n	$n!u^{n+2}$
$t^{a-1}/\Gamma(a), a>0$	u^{a+1}
e^{at}	$\frac{u^2}{1-au}$
te^{at}	$\frac{u^3}{(1-au)^2}$
$\frac{t^{n-1}e^{at}}{(n-1)!}$, $n=1,2,....$	$\frac{u^{n+1}}{(1-au)^n}$
$\sin at$	$\frac{au^3}{1+a^2u^2}$
$\cos at$	$\frac{u^2}{1+a^2u^2}$
$\sinh at$	$\frac{au^3}{1-a^2u^2}$
$\cos\mathrm{h}\, at$	$\frac{au^2}{1-a^2u^2}$
$e^{at}\sin bt$	$\frac{bu^3}{(1-au)^2+b^2u^2}$
$e^{at}\cos bt$	$\frac{(1-au)u^2}{(1-au)^2+b^2u^2}$
$t\sin at$	$\frac{2au^4}{1+a^2u^2}$
$J_0(at)$	$\frac{u^2}{\sqrt{1+au^2}}$
$H(t-a)$	$u^2e^{-\frac{a}{u}}$
$\delta(t-a)$	$ue^{-\frac{a}{u}}$

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